

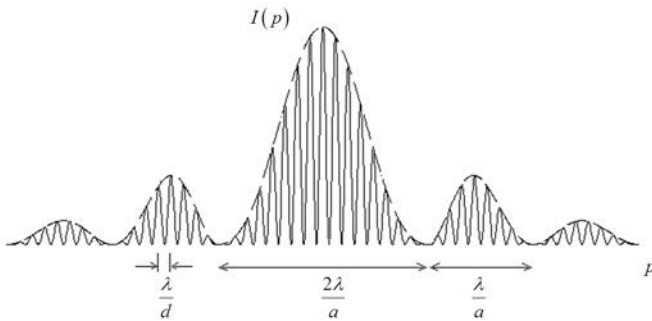
# Discrete-Event Simulation of Quantum Physics Experiments

Kristel Michielsen and Hans De Raedt

In one of his review articles Anton Zeilinger mentioned in 1999 that in former times one could only rely on *Gedanken* (thought) experiments to discuss the foundations of quantum physics, but that because of the tremendous experimental progress in recent years it became possible to base this discussion on actually performed experiments (Zeilinger 1999). Apart from these two options there is a third option to help contribute to this debate, namely performing computer simulations emulating thought and laboratory experiments. For the foundations of quantum physics, this requires a change of paradigm. In traditional, theoretical modeling the behavior of physical systems is described in terms of mathematical models. Usually differential equations, probability theory and so on are used to describe the system and its behavior. In this paper we replace this traditional modeling with a discrete-event simulation in which we model physical phenomena as chronological sequences of events. Although in the discrete-event approach we describe the behavior of systems in terms of simple rules, collectively these systems may exhibit complex behavior. Well-known examples of this approach are the Lattice Boltzmann model, used to simulate the flow of complex fluids, and the cellular automata from Stephen Wolfram (Wolfram 2002).

The community “Collective Evolution,” which promotes thinking outside of the box, published on their website their top three mind-boggling quantum experiments (Walia 2015). The first experiment on their list is the double-slit

experiment with electrons, photons, atoms, molecules, etc., in which the interference pattern is built up event by event. Quantum theory explains this experiment by introducing the concept of particle-wave duality: the property of particles behaving as waves and waves behaving as waves and particles. The second experiment on their list is the delayed choice/quantum eraser experiment. It is often said that this experiment illustrates how what happens in the present can change what happened in the past. The third experiment is an experiment for measuring quantum entanglement, such as the Einstein-Podolsky-Rosen-Bohm experiment for example. In such an experiment it appears that one particle of an entangled pair "knows" what measurement has been performed on the other one and what the outcome of that measurement is, even though there is no known means of information exchange between the particles. Explanations of the observations are sometimes formulated in terms of Einstein's "spooky action at a distance."



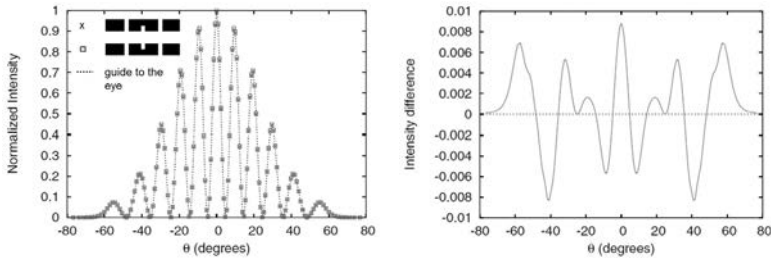
[Fig. 1] Fraunhofer interference pattern  $I(p)$  for a source emitting monochromatic light with wavelength  $\lambda$  and angle of incidence  $\theta_0$  thereby illuminating a plate with two line-shaped slits with width  $a$  and center-to-center distance  $d$ . Here  $p = \sin \theta - \sin \theta_0$ , where  $\theta$  denotes the angle of refraction. The solid line comes from the two-slit interference and the dashed line comes from the single-slit diffraction (see footnote 2).

The single-particle double-slit experiment is one of the most fundamental experiments in quantum physics and thus our focus for this paper. The structure of the paper is as follows. In the first section we briefly recall how to calculate the interference pattern for a two-slit experiment with classical light. We discuss the event-by-event buildup of the interference pattern in two-slit experiments with massive objects (electrons, neutrons) in the second section. As we review in section three, usually quantum theory is used to describe these experiments in terms of single particles, single wave packets or an ensemble interpretation of quantum mechanics. Except for the latter interpretation which is silent on the issue of events, all other

descriptions suffer from some logical inconsistencies. In the fourth section we use a different approach to explain the event-by-event buildup of the interference pattern, namely the discrete-event simulation approach. The last section summarizes our conclusions.

## Two-Slit Experiment with Light

The first two-slit experiment with light was performed by Thomas Young in 1801. In the basic form of this experiment a monochromatic point source is emitting light that falls on a plate with two pinholes that are close together and equidistant from the source. The light passing through the pinholes is observed on a screen placed far behind the plate. The two pinholes act as secondary point sources which emit monochromatic light beams that are in phase. Due to the wave character of the light, light waves passing through the pinholes interfere, thereby producing a pattern of bright and dark bands on the screen, the so-called interference pattern.



[Fig. 2] Left: Simulated interference pattern for two different two-slit configurations. The metal plate with refractive index  $n = 2.29 + 2.61i$  and height  $4\lambda$  has two slits of width  $\lambda$  separated by a center-to-center distance of  $6\lambda$ . In the middle between the two slits is an indent of width  $\lambda$  and height  $2\lambda$ . In one of the two-slit configurations the indent is located at the bottom of the plate (x marks) and in the other configuration at the top of the plate (square marks). The plate is illuminated by light with a wavelength  $\lambda = 500\text{ nm}$ . Right: Difference between the two interference patterns.

From the theory of optics (Born and Wolf 1964) it follows, after performing some relatively simple mathematical calculations using pen and paper, that the interference pattern depends on the wavelength  $\lambda$  and the angle of incidence  $\theta_0$  of the monochromatic light emitted by the two point sources, and on the distance  $d$  between the two sources.<sup>1</sup> In most two-slit

<sup>1</sup> In detail: the intensity pattern is given by  $I(p) = \cos^2(kd p/2)$  with  $p = \sin \theta - \sin \theta_0$  where  $\theta$  denotes the angle of refraction and  $k = 2\pi/\lambda$ .

experiments that are carried out in the laboratory the slits cannot be described as pinholes acting as point sources.

A more accurate representation of the slit is a line-shaped slit. The Fraunhofer diffraction pattern observed on a screen placed at a large distance from an illuminated plate that contains two line slits with width  $a$  and center-to-center distance  $d$  is shown in Fig. 1.<sup>2</sup> But, in laboratory experiments the interference patterns differ from this “ideal” two-slit interference pattern. The cause of these differences is that the assumptions under which the Fraunhofer formula has been derived do not apply: apart from a width slits also have a height and depth and/or the distance between the source and detection screen and/or the source and the plate with the slits might be too small, and/or the slit width is not small enough compared to the source–plate and plate–detector screen distances. Taking into account the experimental details in a derivation of the interference pattern requires more than pen and paper: one has to rely on computer simulations. An example demonstrating that slits cannot simply be replaced by secondary sources and that details in the experimental setup matter for the resulting interference pattern is shown in Fig. 2. It depicts the simulation results for the interference patterns of two different two-slit configurations with an indent between the two slits (De Raedt, Michielsen, and Hess 2012). The results have been obtained by solving the time-dependent Maxwell equation on JUQUEEN (Stephan and Docter 2015), one of the largest supercomputers in Europe. The results show an intensity difference of 0.8%, and this is for an “ideal case simulation.” Even small details in the setup of the devices like indents or other constructive elements obviously matter! A calculation in which the slits were replaced by secondary sources would not show this difference.

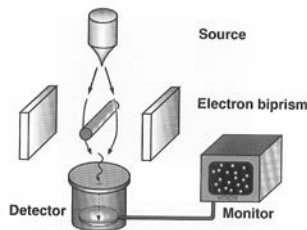
## Two-Slit Interference with Objects: Experiments

These experiments belong to the class of so-called quantum experiments. As mentioned in the introduction, in former times one had to rely on *Gedanken* experiments to study questions related to the foundations of quantum mechanics. In 1964 Richard Feynman formulated a thought experiment for studying the two-slit interference experiment with electrons (Feynman, Leighton, and Sands 1965). The experiment consists of

- 2 The diffraction pattern of one line source with width  $a$  reads  $I(p) = [\sin(kap/2)]/(kap/2)]^2$  (dashed line in Fig. 1). The Fraunhofer formula for the interference pattern observed on a screen placed at a large distance from the illuminated plate that contains two line slits with width  $a$  and center-to-center distance  $d$  reads  $I(p) = \cos^2(kdp/2) [\sin(kap/2)]/(kap/2)]^2$  (solid line in Fig. 1).

an electron gun emitting individual electrons in the direction of a thin metal plate with two slits in it, behind which is placed a movable detector. According to Feynman: (1) one could hear from the detector sharp identical “clicks,” which are distributed erratically; (2) the probability  $P_1(x)$  or  $P_2(x)$  of arrival, through one slit with the other slit closed, at position  $x$  is a symmetric curve with its maximum located at the center of the open slit; and (3) the probability  $P_{12}(x)$  of arrival through both slits looks like the intensity of water waves propagated through two holes, thereby forming a so-called interference pattern, and looks completely different from the curve  $P_1(x) + P_2(x)$  that would be obtained by repeating the experiment with bullets. These observations led Feynman to the conclusions that: (1) electrons arrive at the detector in identical “lumps,” like particles; (2) the probability of arrival of these lumps is distributed like the distribution of intensity of a wave propagated through both holes; and (3) it is in this sense that an electron behaves “sometimes like a particle and sometimes like a wave,”—puzzling behavior for which the concept of particle-wave duality has been introduced. Feynman’s general conclusion about the single-electron two-slit experiment was: “The observation that the interference pattern is built up event-by-event is impossible, absolutely impossible to explain in any classical way and has in it the heart of quantum mechanics. In reality it is the only mystery.”

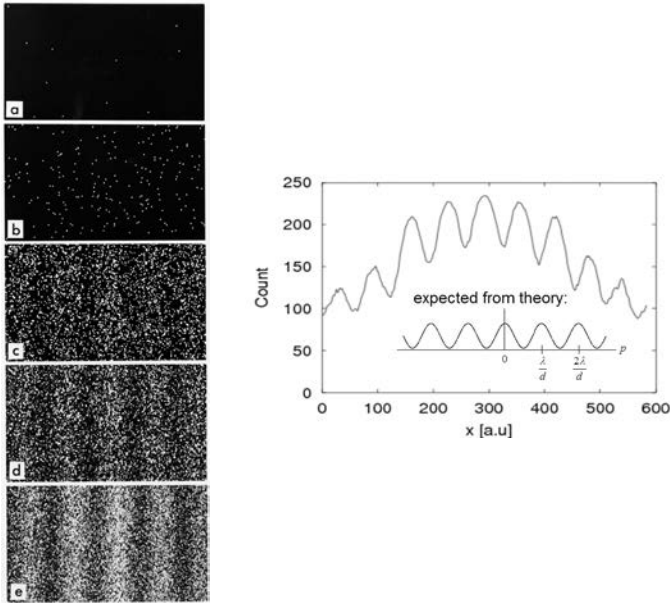
Although Feynman wrote “you should not try to set up this experiment” because “the apparatus would have to be made on an impossibly small scale to show the effects we are interested in,” advances in (nano) technology made possible various laboratory implementations of his fundamental thought experiment. In what follows we discuss a selection of these experiments.



[Fig. 3] Scheme of the setup of the single-electron two-slit experiment (Tonomura 1998).

The first real single-electron interference experiments that were conducted were electron biprism experiments in which single electrons pass to the left or the right of a conducting wire (there are no real slits in this type of

experiment) (see Merli, Missiroli, and Pozzi 1976; Tonomura et al. 1989). A scheme of the setup of the experiment of Tonomura and coworkers is shown in Fig. 3. The setup consists of an electron source, a biprism consisting of a wire and two plates, a detector, and a monitor. In this experiment at any time only one electron travels from the source to the detector. Each electron passes either to the left or the right of the wire before being detected by the detector, which results in a spot on the monitor. After many (about 50,000) electrons have been recorded an interference pattern emerges. Hence, although there is no interaction between the electrons they build up an interference pattern one by one.



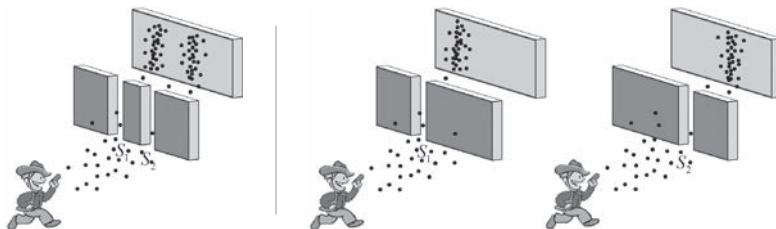
[Fig. 4] Left: Recordings of a single-electron double-slit experiment performed by Tonomura and coworkers showing the buildup of an interference pattern with an increasing number of detected electrons. Numbers of electrons are 11 (a), 200 (b), 6,000 (c), 40,000 (d), 140,000 (e) (Tonomura et al. 1989). Right: Final interference pattern. The inset shows the interference pattern expected from theory.

The buildup of the interference pattern is depicted in the left panel of Fig. 4. If the number of detected electrons is small, then the single spots on the monitor screen seem to be positioned randomly; after a larger number of electrons have been detected stripes are formed. From these observations one could conclude that electrons are detected one by one as particles. The right panel of Fig. 4 shows the intensity pattern obtained from the stripe

pattern at the end of the experiment. The intensity pattern differs from what would be expected from theory for the ideal experiment but it does show interference. This interference pattern is often said to be formed when electron waves pass both sides of the wire at the same time. Hence, it is concluded that electrons in this experiment show both particle and wave character.

Rather recently, another realization of Feynman's thought experiment has been performed making use of a plate with two slits instead of an electron biprism (Bach et al. 2013). In this experiment a movable mask is placed behind the double-slit structure to open/close the slits. Unfortunately, the mask is positioned behind the slits and not in front of them, so all the electrons encounter the double-slit structure and are filtered afterwards by the mask. One could therefore argue that as of 2017 Feynman's thought experiment has still not been performed.

Interference experiments can also be performed with "objects" other than electrons. One example is the single-photon interference experiment of Jacques and coworkers (Jacques et al. 2005). This experiment is similar in spirit to that of Tonomura and coworkers except that photons are used instead of electrons. The experimental setup consists of a single-photon source, a prism with a very shallow angle that splits the beam (a so-called Fresnel biprism), and a detector. After many single detection events an interference pattern is observed. Another example is the single-neutron two-slit experiment of Zeilinger and coworkers (Zeilinger et al. 1988; Gähler and Zeilinger 1991), which is also of the same type as Tonomura's experiment. The setup consists of a neutron source, a wire, and two glass plates. As in the other experiments, care is taken that only one "object," in this case a neutron, at a time is ever traveling through the setup so that there can be no interaction between the neutrons. Also in this experiment, after many neutrons have been detected one by one, an interference pattern is seen. In this experiment the dimensions of the double slit are measured with an optical microscope and are also obtained by fitting curves to the experimentally measured interference pattern. Both methods give different results for the dimensions of the double slit, showing that the reality of an actual lab is much more complicated than the world of the *Gedanken* experiment. It is quite common practice to first extract the double slit dimensions from the experimental data by fitting them to Fraunhofer-like diffraction formulas and then comparing the measured interference pattern to the one obtained by numerical simulation with the extracted double slit dimensions.



[Fig. 5] Left: Two-slit interference experiment with bullets. Right: Two one-slit experiments with bullets.

The original goal of the two-slit experiments was to demonstrate that not only waves but also “objects” (particles) can interfere. This original goal has shifted to obtaining interference with “objects” that are as large as possible, such as large organic molecules.

## Two-Slit Interference with Entities: Description-Explanation

From now on we will call electrons, photons, neutrons, atoms and molecules “entities.” It is important to stress that entities are indivisible units; in other words, they cannot split. In the two-slit interference experiments one click of the detector is associated with one entity arriving at the detector. Only after many single detection events does an interference pattern emerge. The interference patterns can be fitted by wave diffraction theory. The so-called dual particle-like and wave-like behavior of the entities can be explained in different ways. In what follows we discuss some of these explanations.

### Entities are Particle Like

If the entities are particle like, then the two-slit experiment is well described by Feynman’s interference experiment with bullets. Fig. 5 shows such an experiment in which we replaced Feynman’s machine gun with a shooting cowboy. The cowboy shoots one bullet at a time towards a front wall that has one or two openings. The position of the cowboy with respect to the front wall is the same for each experimental setting. The positions of the openings in the front wall in the single-slit experiments correspond to their respective positions in the front wall in the double-slit experiment. In cases where the cowboy is shooting towards the wall with two openings labeled  $S_1$  and  $S_2$ , the bullet passes through one slit or the other and arrives



at a certain position on the rear wall that serves as a shield (a bullet is indivisible but one cannot observe through which slit it passes) or is stopped by the front wall. In cases where the cowboy is shooting towards one of the front walls with only one opening, labeled  $S_1$  or  $S_2$ , the bullet passes through this single slit and arrives at a certain position on the rear wall or is stopped by the front wall. After the three experiments are finished one observes that the bullet hole pattern of the two-slit experiment is equal to the overlay, the sum of the bullet hole patterns of the two one-slit experiments. There is no interference. And if the bullets had been electrons, this is in contradiction to the observations made in the Tonomura experiment!

Any probabilistic theory, hence also quantum theory, describing these experiments postulates the existence of an underlying probabilistic process that determines the patterns with which the bullets will be observed. However, in these probabilistic descriptions the probabilities are conditional on the fact that a slit is open or closed—conditional probabilities with different conditions cannot be added (Ballentine 2003)<sup>3</sup>. In the case at hand, probability theory does not allow the addition of the probabilities of the single-slit shootings in any theoretical description of the process! Nevertheless, Feynman (and many others) did so because he simply forgot about the conditions.

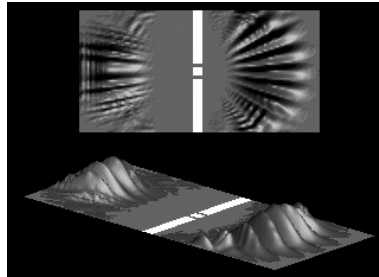
Hence, although the naïve conclusion that the observed interference patterns in two-slit experiments with “entities” cannot be obtained with entities passing one by one through the double-slit device might at first sight seem correct, we will demonstrate in the section entitled “Two-slit

- 3 In the two-slit experiment with bullets one may get the impression that it is allowed to add conditional probabilities with different conditions, which are derived from the experimentally observed frequencies:  $\sum_x P(x|S_1, S_2, Z) = 1$  is equal to  $\sum_x P(x|\overline{S_1}, S_2, Z) + \sum_x P(x|S_1, \overline{S_2}, Z) = 0.5 + 0.5 = 1$ . Here,  $x$  denotes a position on the detection screen,  $S_j(\overline{S_j})$  corresponds to an open (closed)  $j$ -th slit ( $j = 1, 2$ ) and  $Z$  denotes all other identical conditions under which the three different experiments are carried out (e.g. the position of the shooting cowboy, the positions of the slits, the cowboy shoots half of the bullets in the direction of each slit, ...). In general, one is not allowed to add conditional probabilities with different conditions, as can be seen from considering the following three experiments with variables (negations are represented by an overbar)  $R$  denoting that it rains,  $W$  representing that one gets wet from rain only,  $U$  denoting the fact that one has a very large umbrella that one uses not to get wet from rain, and  $Z$  are all other identical conditions in the different experiments: (1) it rains and one does not have a very large umbrella, (2) it does not rain and one has a very large umbrella, and (3) it rains and one has a very large umbrella. In this example  $P(W|R, U, Z) = 0$  can definitely not be equal to  $P(W|R, \overline{U}, Z) + P(W|\overline{R}, U, Z) = 1 + 0 = 1$ .

interferences with entities: discrete-event simulations” that this is in fact not the case.

### Entities are Wave Packets (Wave Like)

Under this assumption the picture is that a wave packet with a size larger than the center-to-center separation of the slits plus the slit width impinging on a double-slit device interferes with itself. According to quantum theory the time evolution of the wave packet is governed by the time-dependent Schrödinger equation  $i\hbar\partial\psi(x, t)/\partial t = H\psi(x, t)$ , where  $H$  denotes the Hamiltonian of the two-slit system,  $\psi(x, t)$  represents the wave function of the complete system, and  $\hbar$  is Planck’s constant. Fig. 6 depicts a snapshot of a movie showing the time evolution of a Gaussian wave packet impinging on a double-slit device and thereby being partly reflected and partly transmitted. However, although the initial wave packet is split in two parts, at any time there is only one wave function,  $\psi(x, t)$ . What is actually shown in Fig. 6 is the intensity  $|\psi(x, t)|^2$ , which according to the Born rule gives the probability of finding the entity at position  $x$ . The “large” transmitted part of the wave packet emanating from the double slit reaches the detection screen. Thinking of the laboratory experiments, one expects the wave packet to produce one single spot on the screen because in experiments one does not observe the single interference of one entity.



[Fig. 6] Intensity of a Gaussian wave packet of width  $\sigma = 10\lambda$  reflected and transmitted by a wall with two slits in it (Michielsen and De Raedt 2012). The thickness of the wall is  $\lambda$ . The slits have a width  $a = \lambda$  and a center-to-center distance  $d = 4\lambda$ . The initial wave packet moves from the left to the right.

Note that what should happen to the reflected part of the wave packet that is moving in the direction of the source is unclear. Heisenberg introduced in 1927 the reduction and Bohm in 1951 the collapse of a wave function to explain how a single entity represented as a wave packet can give rise to a single spot on a screen. However, this does not explain the event-by-event

buildup of the interference pattern, i.e. the coordination between the detection events resulting from many “large” wave packets arriving at the detection screen. How should one explain this—by Einstein’s spooky action at a distance? After almost 100 years, the collapse of the wave function remains elusive and does not provide a rational explanation of the observations in a two-slit experiment with single entities.

### Ensemble Interpretation of the Interference Pattern

According to Einstein, “The attempt to conceive the quantum mechanical description as the complete description of individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems” (Einstein 1949). In other words, one should not try to explain *individual* events using quantum theory.

Interpreting the wave packet (see e.g. Fig. 6) as one probability wave, representing the collection of all entities that is propagated through the double-slit device according to the rules of quantum theory, leads to an interference pattern that is similar to the final one observed in a laboratory experiment. However, this ensemble interpretation gives no clue about how to get from the final probability distribution to the detection events observed in the experiment. Events can simply not be “derived” from quantum theory (or from probability theory). Hence, the ensemble interpretation cannot explain the event-by-event buildup of the interference experiment.

Clearly we have here a dilemma. If, as Einstein said, we refrain from making statements about individual events, quantum theory is logically consistent. For atomic spectra quantum theory even gives a quantitative description. However, for the outcome of single-entity interference experiments or of experiments in which entanglement is involved quantum theory often only gives a qualitative description. This raises the question: how can it be that we have a very successful theory (quantum theory) that says nothing about the individual observations that make up the collective which the theory (quantum theory) describes very well?

As quantum theory cannot say anything about individual observations, another question that arises is whether it is possible to conceive ways of producing the kind of events that we observe in experiment directly, without referring to the concepts of quantum theory. The answer to this

question is affirmative. For many of the so-called fundamental quantum physics experiments it is possible to construct a fairly universal computer simulation model that reproduces the results of all these experiments through discrete-event simulation, without solving wave equations and the like: for example see (De Raedt, De Raedt and Michielsen 2005; Michielsen, Jin and De Raedt 2011; De Raedt, Jin and Michielsen 2012; Michielsen and De Raedt 2014).

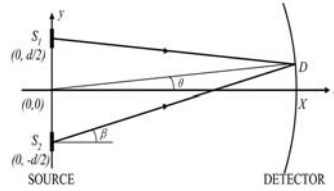
## **Two-Slit Interferences with Entities: Discrete-Event Simulations**

Discrete-event simulation is a very general form of computer-based modeling. It provides a flexible approach to represent the behavior of complex systems in terms of a sequence of well-defined events; that is, operations performed by processors on entities of certain types. The entities themselves are passive, but they have attributes that affect the way they and their attributes are handled by the processors. Typically, many details about the entities are ignored. The events occur at discrete points in time. The system does not change between consecutive events. Discrete-event simulation is used in a wide range of health care, manufacturing, logistics, science, and engineering applications. We use discrete-event simulation to model various single-entity experiments relevant to the foundations of quantum physics.

In contrast to standard mathematical modeling, discrete-event simulation starts directly from experimental observations. In discrete-event modeling one searches for a logically consistent, cause-and-effect description of the definite results (the events) that constitute the experimental facts. Hence, one goes from events to probabilities and not vice versa. Therefore, the algorithm in a discrete-event simulation cannot refer to a probability distribution to produce the events. The resulting model may or may not fit into classical (Hamiltonian) mechanics. As in discrete-event modeling one starts from human perception, then goes to events, and finally arrives at a quantitative description, there is no need for an “objective” mathematical world picture. In our discrete-event simulation of single-entity experiments quantum theory emerges through inference from the events. We illustrate this with two examples related to two-slit interference with single photons.

### Two-Slit Experiment with Two Beams (Two Sources)

From the theory of optics it follows that Young's double-slit experiment can be simplified to a two-beam experiment by replacing the two slits with two virtual sources. The two-beam experiment allows us to study interference in its most pure form because in contrast to the two-slit experiment the phenomenon of diffraction is absent. A time-resolved two-beam experiment has been performed in the laboratory (Garcia, Saveliev, and Sharonov 2002).

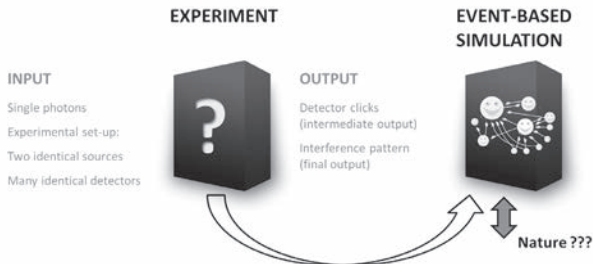


[Fig. 7] Schematic diagram of a two-beam experiment with light sources  $S_1$  and  $S_2$  of width  $a$ , separated by a center-to-center distance  $d$ . Both sources emit coherent, monochromatic light. The angles of emission  $\beta$  are uniformly distributed. The light is recorded by detectors  $D$  positioned on a semicircle with radius  $X$  and center  $(0, 0)$ . The angular position of a detector is denoted by  $\theta$ .

A schematic setup of the two-beam experiment with coherent, monochromatic light sources is shown in Fig. 7. In a single-photon version of the experiment the single-photon sources emit photons one by one. In the discrete-event model of this single-photon experiment entities are created one at a time by one of the sources (creation events) and are detected by one of the detectors forming the detection screen (detection events). We assume that all detectors are identical and cannot communicate with each other. We also assume that there is no direct communication between the entities (there is always only one entity between the source and the detector plane). Hence, the discrete-event model is *locally causal* by construction. If the entities build up an interference pattern one by one, then the interference pattern can only be due to the internal operation of the detectors, which has to be more complicated than just counting the incoming entities. We disregard the option that a similar interference pattern can be obtained by adding the detection events from a huge set of detectors that each only detected one entity. We do not consider this option, which is based on the statistical property of quantum theory, because there is no experimental evidence that replacing detectors after having detected an entity and then combining all these detection events indeed results in an interference pattern. The discrete-event simulation is

based on observations made in laboratory experiments and not on hypothetical theoretical considerations.

Fig. 8 illustrates the general idea behind the discrete-event simulation approach. Simple rules define discrete-event processes that may lead to the behavior observed in experiments. The basic strategy in designing these rules is to carefully examine the experimental procedure and to devise rules such that they produce the same kind of input and output data as those recorded in the experiment. Evidently, mainly because of insufficient knowledge, the rules are not unique. Hence, the simplest rules one could think of can be used until a new experiment indicates otherwise. Obviously, the discrete-event simulation approach is concerned with what we can say about these experiments but not what “really” happens in nature.



[Fig. 8] Schematic of the working principle of the discrete-event simulation approach. The first step consists of a detailed analysis of the experiment. Information about the input, such as characteristics of the source(s) and all other components in the experimental setup, and the output, such as the detector clicks (intermediate output) and the interference pattern or correlation (final output), including the data analysis procedure, is collected. It is assumed that it is not known how the input is transformed into the output. In a second step the “black box” that connects input and output in the experiment is replaced by a set of simple rules that transform this input into the same output. The frequently asked question about whether the rules describe what is going on in nature cannot be answered because the information necessary to answer this question is lacking.

The general picture in the discrete-event approach is that the entities are seen as messengers that carry certain messages, such as polarization, time, frequency, and so on, and run around in the experimental setup. The optical components in the experiment, in this case the two sources and the detectors, are seen as processors that interpret and manipulate the messages. It is important that the messengers do not communicate directly, only indirectly through the processors. This complies with the notion of local causality.

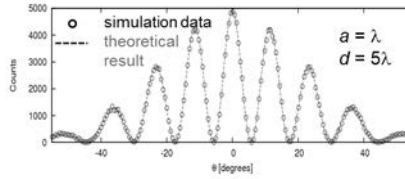
We now specify in a bit more detail the set of simple rules for simulating the two-beam, single-photon experiment. More detailed information can be found elsewhere (De Raedt and Michielsen 2012).

- Photons: The photon is regarded as a messenger carrying a message  $\vec{e}(t) = (\cos 2\pi f t, \sin 2\pi f t)$  that is represented by a harmonic oscillator which vibrates with frequency  $f$  (representing the “color” of the light). The internal oscillator is used as a clock to encode the time of flight  $t$ , which depends on the source–detector distance. Think of this message as the hand of a clock that rotates with frequency  $f$ .
- Source: The source creates a messenger and waits until its message has been processed by the detector before creating the next messenger, so that there can be no direct communication between the messengers. When a messenger is created its time of flight is set to zero.
- Detector: We describe the model for one of the many identical detectors building up the detection screen. These detectors operate independently from each other. Detectors are very complex devices. In its simplest form, a light detector consists of a material that can be ionized by light. This produces a signal, which is amplified. In Maxwell’s theory, the interaction between the incident electric field and the detector material is the result of a coupling of the oscillation of the incoming photon with the polarization of the detector material due to the photons that came in previously. In cases where incoming photon and remaining polarization are in phase—in the same state of oscillation—the detector is likely to click; in cases where they are out of phase, no click will occur.<sup>4</sup> If the “memory” of the detector is good enough and if there are enough messengers, the event-based simulation generates the interference pattern

- 4 In more detail: the interaction between the incident electric field  $\vec{E}$  and the detector material takes the form  $\vec{P} \cdot \vec{E}$ , where  $\vec{P}$  denotes the polarization vector of the material. In the case of a linear response of the material  $\vec{P}(\omega) = \chi(\omega)\vec{E}$ , where  $\chi$  denotes the electric susceptibility of the material and  $\omega$  is the frequency of the impinging monochromatic light wave. In the time domain this relation expresses the fact that the material retains some memory about the incident electric field,  $\chi(\omega)$  representing the memory kernel. In the discrete-event model, the  $k$ th message in the form of the two-dimensional vector  $\vec{e}_k(t) = (\cos 2\pi f_k t, \sin 2\pi f_k t)$  is taken to represent the elementary unit of electric field  $\vec{E}(t)$ . The electric polarization  $\vec{P}(t)$  of the material is represented by a two-dimensional vector  $\vec{p}_k$ . Upon receipt of the  $k$ th message by the processor modeling the detector this vector is updated according to the rule  $\vec{p}_k = \gamma \vec{p}_{k-1} + (1 - \gamma) \vec{e}_k$  where  $0 < \gamma < 1$  and  $k > 0$ . After updating the vector  $\vec{p}_k$ , the processor uses the information stored in this vector to decide whether to generate a “click.” As a highly simplified model, we let the processor generate a binary output signal  $S_k$  using the intrinsic threshold function  $S_k = \Theta(\vec{p}_k^2 - r_k)$ , where  $\Theta(\cdot)$  denotes the unit step function and  $0 \leq r_k < 1$  is a uniform pseudo-random number. For  $\gamma \rightarrow 1^-$  and a large enough number of messengers we recover the interference pattern from wave theory.

that we know from wave theory. This detector is a kind of adaptive machine that “learns” from the incoming entities.

The whole algorithm is very simple and does not require a lot of computer power: a personal computer suffices. Fig. 9 shows a comparison of the simulation results from about six million entities with the theoretical result  $I(\theta) = A[\sin((a\pi \sin \theta)/\lambda)/(a\pi \sin \theta/\lambda)]^2 \cos^2((d\pi \sin \theta)/\lambda)$  obtained from a straightforward application of Maxwell’s theory in the Fraunhofer regime. As can be seen, the agreement is excellent. The agreement is not only perfect for this parameter set but also for many others (Jin et al. 2010).



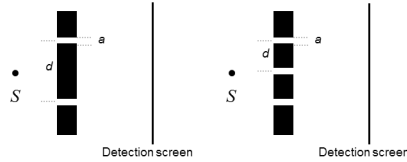
[Fig. 9] Detector counts as a function of the angular detector position as obtained from event-by-event simulations of the two-beam interference experiment depicted in Fig. 7. The sources, emitting particles, are slits of width  $a = \lambda$  ( $\lambda = 670$  nm), separated by a distance  $d = 5\lambda$  and the source–detector distance  $X = 0.05$  mm. A set of 1,000 detectors is positioned equidistantly in the interval  $[-57^\circ, 57^\circ]$ , each of them receiving on average about 6,000 photons. In the simulation model  $\gamma = 0.999$ .

### Multiple-Slit Experiment with Slit Device

We consider the interference experiments with two-slit and three-slit devices as depicted in Fig. 10. In contrast to the two-beam experiment, in these experiments not only interference but also diffraction occurs. In the discrete-event model of these experiments the rules for the photons and source are the same as the ones used to simulate the two-beam interference experiment. As we may assume that in this case the multiple-slit device, and not the detectors, causes the diffraction and interference, the adaptive machines modeling the detectors are replaced by counters that simply count each incoming messenger.

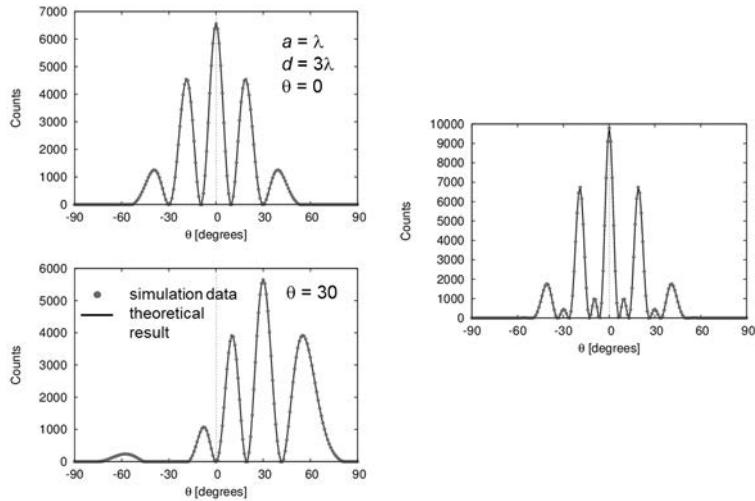
An adaptive machine models the multiple-slit device. An entity follows the classical trajectory in the multiple-slit device thereby possibly transferring momentum to the multiple-slit device. Hence, the multiple-slit device is modified by the passing entity and as a result each passing entity experiences a slightly different multiple-slit device. Thus the multiple-slit device is a kind of adaptive machine that “learns” from the incoming entities.





[Fig. 10] Setup for a single-entity experiment with a two-slit device (left) and a three-slit device (right).

Fig. 11 shows some simulation results for entities impinging on a two-slit device at normal incidence ( $\theta = 0$ ) and under an angle of incidence ( $\theta = 30^\circ$ ). Also a result for a three-slit device on which entities impinge at normal incidence is shown. The simulation results are compared with the theoretical results in the Fraunhofer regime and again perfect agreement is found.



[Fig.11] Detector counts as a function of the angular detector position as obtained from event-by-event simulations of the multiple-slit interference experiments shown in Fig. 10. Left: Two-slit device, Right: Three-slit device.

## Conclusions

The discrete-event simulation method models physical phenomena as chronological sequences of events. The events in the simulation are the action of an experimenter, a particle emitted by a source, a signal detected by a detector, a particle impinging on a material, and so on. These are the events that are extracted from a thorough analysis of how the experiment

is performed. The next step, and this is the basic idea in the approach, is to invent an algorithm that uses the same kind of events (data) as in experiment and reproduces the statistical results of quantum or wave theory without making use of this theory. Discrete-event simulation successfully emulates single-entity experiments (so-called quantum experiments) demonstrating interference, entanglement, and uncertainty. By construction, the discrete-event approach is free of logical inconsistencies.

In principle, a kind of Turing test could be performed on data coming from a single-entity interference experiment performed in the laboratory and on data generated by the discrete-event simulation approach. This test would lead to the conclusion that both data sets look quite similar. The observer would be quite puzzled because this type of laboratory experiment is often classified as “quantum” yet no quantum theory is used in the discrete-event simulation.

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## Discussion with Kristel Michielsens and Hans De Raedt

Eric Winsberg: So, I have two questions—one quick sort of specific question and a more general one. The quick question is: there was this experiment done a couple of months ago, I think, which claimed it closed the Einstein–Podolsky–Rosen (EPR) loopholes or whatever. Can you guys do that within your paradigm here?

Kristel Michielsens: We have simulated EPR experiments, yes.

EW: But the one that was just done a couple of months ago, that supposedly closed the loopholes or whatever?

KM: There are two different approaches. On the one hand we can simulate various experiments. For this particular experiment we have to study how to implement it. That's one thing. On the other hand there is a fundamental problem with this type of experiment and for us it doesn't matter whether all the loopholes are closed or not because there will always be one remaining. That's simply because one cannot perform the thought experiment as it was originally designed. Hence, these are two different things. But if one performs an Einstein–Podolsky–Rosen–Bohm (EPRB) experiment and finds a violation of a Bell-type inequality then we can simulate it. For example, we have simulated the single-photon EPRB experiment performed by Gregor Weihs in Vienna. We have also simulated the EPR experiment with neutrons. So those two EPR experiments we already simulated—but of course, people come with more and more experiments.

EW: Okay, here's my more sort of philosophical question. There are a number of ways of thinking about what the puzzles in quantum mechanics are. One way of thinking about it that I sort of find useful is that what seems to be wrong in a way with the conventional presentation of quantum mechanics is that it gives us two different laws of evolution. It says there's the time-dependent Schrödinger equation, which evolves the wave function until you measure it and then there is a collapse. Why is there a collapse when you measure it? What's so special about measurement? Shouldn't measurement be described by the same theory that describes the evolution of the rest of the world? Why do measurement devices obey different laws than the rest of the world? It seems to me that one necessary condition for having a kind of adequate foundational story about what's going on

in quantum mechanics is to not have that difference between how the world behaves and how detectors behave. But it seems to be built into your way of doing things that there...

KM: There's no difference. In our approach there is no difference between the detectors and all the...

EW: But don't you have different rules for entities and detectors and such? I thought that was kind of the...

KM: No, because...

EW: I mean, one way of thinking about it is this: in a way, whatever kind of representational system one has for the world, whether it's differential equations or event simulations or whatever it is, what one would like at the end of the day is one theoretical apparatus for quantum systems and for measurement systems and not to treat them separately.

KM: But in our approach they are not treated separately. We are always designing consistent models. It depends a little bit on the experiment you're looking at. Sometimes we encounter an experiment for which we have to build in new features. This could be a new apparatus for example, or it can be like as shown here, in the two-beam and two-slit experiment. In the case that you only have two sources and a detector, the detector has to be special, you could say. It needs to have some rules.

If we have this other device between the source and the detector, this two-slit device, then we can say that this two-slit device plays a special role and that we can take a very simple detector, which is simply counting every incoming entity. What we mean by saying that the simulation model has to be consistent is that if we take our more complex detector and put it behind the two-slit device, we can still obtain an interference pattern. The idea is that we cannot know beforehand how complicated the device needs to be for simulating all kinds of experiment. Another very simple example is a beam splitter. One can make the model very simple, and say I observe 50% of the entities is going left and 50% is going right—I can just put a random number generator in place of the beam splitter: half is going left and half is going right. Fine. If one is going to make a Mach-Zehnder interferometer with this type of beam splitter, it's not going to work. In that case one needs something more complex for the beam splitter.

From then on we use the more complex model for the beam splitter and use it to construct other experiments. We do the same in modeling other devices. So what we do is make a toolbox. We want the toolbox to be consistent. In the end the toolbox should be such that if one is designing an experiment one should be able to say I need this and this and this apparatus, so I go to the toolbox and take all the corresponding components, put them together, and simulate the experiment. That is our approach. In that sense there is no big difference between simulation and experiment. We make no distinction between classical and quantum. By the way, one can indeed say that one should include the detector in the quantum theoretical description. One can do that no problem because then one has one big quantum system. But, this does not solve the problem. Quantum theory describes the whole system, the whole experimental setup, including in principle the detector. But, it does not help, where does one stop?

Hans De Raedt: So, I think the final problem is the event. One has to explain the event. The fact that our brain somehow registers an event means that in the end one has to put a measurement system in our brain, if one goes with this logic of always extending quantum theory to incorporate more and more and more.

EW: Right, I mean there are various approaches to this, one is to think that if you get enough stuff in the same place it collapses as a law or, you know...

HDR: There are difficulties there. So if you say the collapse, we have to evoke the collapse, then the collapse is outside of quantum theory. The formula is not quantum theory, it's something external. It's fine, but in the end if you do the logic you have to say everything collapses in my brain. Not only in yours but also in mine. In everyone's brain. Of course we can believe that, but the question is not whether it's true or not: the question is whether there is a more rational explanation to it.

Lukas Mairhofer: Let's put it this way, Karen Barad tells us that Niels Bohr told us that if you look at an interference experiment you somewhere have to make a cut between your observed system and your observing system. Where you make the cut is kind of arbitrary but it determines what the result of your observation will be. For me, what you told us so much resembles this that for me it's really hard to believe that you treat quantum systems and classical systems alike. Because you showed us that the adaptive system can be the screen or the diffraction element, and I would claim that it should be possible to make

the entity the adaptive system. Just that you're able to change the functions of different parts of your experimental system—isn't that something that is so inherently quantum and that is so much not there in the classical world?

KM: First of all I would say there is no quantum world and there is no classical world. The only thing we can do is give a description of the world. This way of describing is just the same technique I use here to simulate these so-called quantum experiments. One is always talking about quantum experiments but the question is, are they really quantum? What does it mean? So, that's another question. Actually, using the same methods and apparatuses we simulate classical optical systems. We can simulate the Brewster angle single photon by single photon. Where then is the quantum?

LM: But can you do it with classical billiard balls like atoms?

KM: Yes. On the computer we can. But these are just simple models for what is going on. It gives a description in terms of...

LM: I think my problem is that I have the feeling that your whole epistemic approach is not classical. Because ascribing these adaptive functions, or being able to ascribe this adaptiveness to an arbitrary part of the system, is already something where the line between the observer and the object is getting so blurred and so on that in a Newtonian world this somehow feels very awkward for me. But okay.

KM: Okay. Hans De Raedt, do you have a comment?

HDR: What's so special about Newton?

LM: I just want to say that to me it seems that it's not Newtonian. It's not the classical epistemic approach.

HDR: Yes, but don't mix classical with Newton. Of course it's not classical Hamiltonian.

KM: It's not Hamiltonian mechanics, but in a sense it's maybe better to think outside of physics like for the other examples I have given. It's a methodology applied to physics but maybe it's less strange if you forget about...

LM: I don't find your approach strange. I just would find it strange to link this approach to a classical epistemic world view where there is a strict separation of the observer and the observed system. Because what

you described is so completely different from that. That's all I wanted to say.

Mira Maiwöger: If you would want to simulate an experiment that throws apples through two slits, what would you need to change in order to get the two Gaussian probability distributions overlapping? The distribution that one can observe when one throws lots of apples through two slits?

KM: This is also a matter of dimensions and parameter values. If we do these two slit experiments, think about the dimensions. We have these rules and then it still fits.

HDR: In this particular case you just turn off the adaptiveness of the machine. That's it. Then it simply makes straight trajectories. In a sense you turn off the interaction of the entity and the slit.

KM: You have this parameter gamma there, so you have a range of possible values. If one goes to the wave description then we take gamma close to one and otherwise close to zero...

HDR: That is also what it is in Feynman's picture: it thinks of bullets going through the slit and the bullet and the slit. I mean one could take away the slit and just shoot the bullet in a narrow region and one would have the same answer. So, if we do this in the simulation, say switch off the interaction between the slit and the object, then bullet behavior is observed. So, essentially that is the rule. If one removes the adaptiveness it behaves as classical Hamiltonian mechanics.

Stefan Zieme: I think I have the same question that has been asked several times before—just to be sure that I got it right. You have a local description of your entities in your simulation? My first question would be how do you cope with Bell's inequality, and didn't you just shift everything you did into the detector? That would be my first thought. If you didn't, how would you then cope with Bell's inequality? I would find that rather strange, especially in regard of your EPR-Bohm experiment, because it's hard to see what you are simulating—that would be the first question that comes to my mind. If you talk about local entities in your simulation I have the impression you just shifted the problem to the detector. By training I have to say this; it's not that I'm convinced of it, but my training...



KM: You have to ask this question. Then I would say it's hard to ask about simulating Bell's inequality experiment, because we really have to see how this experiment is performed.

SZ: Like I think Clauser in 1972 was the first one to come up with this idea. I don't know anything about that, only very little. But I wondered how yours compares with that one?

KM: I will tell you what the most important ingredients are. We all have in mind the thought experiment of EPR, so a source sending pairs of particles. One particle is going to the left, the other one is going to the right. If one is up then you detect the other one as down. So that's one thing, but now one is going to do an experiment. This situation is not so ideal because one has to, in the end, identify pairs. If one looks closely at the experiments, it depends on how it's done, but in most of them time is needed in order to determine whether the particles belonged to a pair. So, there is some coincidence time needed in order to determine whether one has pairs. This already tells one that if one does a simulation, time is an important ingredient, which is not present in quantum theory. One then has to see how to simulate the experiment as the experimenters do it. This also means that one has to do the data analysis in the same way as the experimenters do it. What they do is choose a certain time window themselves. If we include all these ingredients in our simulation then although we have a local method, we correlate the data based on time stamps and by comparing time differences to a time window. So...

HDR: Maybe I may add here. In this particular case, this is the simplest simulation you can do from our perspective in the sense that you do not even need adaptive machines for it. So the only thing you have to do is...

KM: Is you have a source.

HDR: One has the source. One looks at what the experiment really entails, not at some idea that people have about the experiment. One really looks at how the experiment is being done, one puts all these things together, and one makes a simulation of it and it simply reproduces everything.

KM: In this case it's simple. You have a source emitting pairs and you have a detector that simply counts. Everything is counted.

SZ: So you will measure something that is bigger than two?

HDR: Absolutely.

SZ: And you have a local description? There's something contradictory. I don't know where to put the contradiction yet for me.

KM: No, it's even stronger. What we observe is a correlation that exactly corresponds to the one of the singlet state. So we do not only find a violation, but it is two times the square root of two. Another thing that we find, and which is usually not shown in the experiment, is that the single particle expectation value is zero and does not depend on the setting.

EW: So this is just by way of giving you a little bit of an idea of what might be going on here. There's a sort of long tradition of studying the Bell-type experiments by looking at the detector efficiency. If you rule out the assumption—all the analyses of these experiments relies on the assumption that when the detectors fail to detect that's a random event—if you give that up, you have a lot of wiggle room, and something like that is going on here I assume, but I'm not sure.

KM: Some filtering is going on, which...

HDR: Mathematically speaking, everybody refers to Bell's inequality but one can also look at the experimental situation, which by necessity requires measurement of times. Then generalize Bell's inequality to this situation. The inequality changes and this new inequality one can never validate—never. The limit is not two, the limit is four. This has been done by many people, but it's hardly mentioned in literature. So nobody seems to care.

KM: So one has to look at the correct inequality.

Martin Warnke: I would like to ask a question to everybody, not just the two of you. Could all this puzzlement we're now experiencing collectively, could that stem from the fact that we are newly coming down from the Platonic heaven of ideas to a very, very concrete description of what's actually happening? Could that be the media effect that we always look for? Might computer simulations have in this case the effect that you could deviate from very tough idealizations to a very concrete description? Might that be the difference? It seems that to me, but I'm not sure about it.

HDR: I certainly agree. I think as KM said the basic starting point is perception, not some idea we have about the world.

KM: So, not a mathematical model that is already based on many assumptions and simplifications.

MM: My question is could you have conceived a Bell experiment if not for this ideal, if not for these ideas of quantum physics? Could this experiment have been done? I think it's an interaction of course; this description is really concrete, but would there be experimental evidence of a Bell-type experiment without the idea of quantum physics being there?

HDR: If I remember the history of Bell's work well, Bell set up this inequality to prove quantum theory wrong, not to prove it right. So... no, no... that is what was made afterwards.

KM: Afterwards, not originally.

HDR: Bell was a strong believer in Bohmian theory and he wanted to show, that was his intention, he wanted to show that quantum theory was wrong. The experiment turned out to violate the inequality and then people started to change... you can look up the history. This has been lost somehow.

EW: You're right that Bell was a Bohmian, that's absolutely right, but Bohm's theory is nonlocal and so what Bell was out to prove was that there couldn't be a local rival to Bohmian mechanics.

HDR: Maybe we're not going to discuss these kinds of things.

Arianna Borrelli: I just wanted to say something on the subject of this media effect. I think here you can really see the power of a very powerful medium—mathematical formalisms. Because here the whole discussion in my opinion has very strongly been framed in terms of quantum mechanics versus classical mechanics. Is it the equations of quantum mechanics or the classical ones that are true? This is actually, from what I understood from the work that was presented here, not the point. This is more like you have the experiment, you have the perception. We have some clicks. We have some different mathematical formulas. Quantum mechanics, also classical mechanics, but that's not relevant in this context. Then we had maybe something else, something different, computer simulation. And this is the tension that is being presented here. I think it's sometimes difficult to approach, to frame the question in these terms, without immediately jumping and looking at what other mathematical formalisms are there. Of course all of these discussions could not have come up without quantum theory

being there. That's clear—it would be crazy. That's not the issue, I just wanted to highlight this.

KM: Indeed, I agree. There is too much classification into classical, quantum, but we only look for an explanation or for a description so to speak. That's the only thing. Indeed.