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Computational Visualistics and Picture Morphology – An Introduction

Abstract

Pictures have to be formalized digitally in an adequate manner when computer scientists are to work with them. It is mainly the relevant physical properties of the corresponding picture vehicle that have to be considered in that formalization: that is, the picture syntax. The present special issue of IMAGE deals in particular with morphological questions taking the specific, formalizing perspective of computational visualistics. It is also intended as the attempt to offer a clear and easily understandable summary of the state of the art of research on picture morphology in computational visualistics for picture scientists of the other disciplines. As an introduction, the relations between computer science, general visualistics, syntax studies, and morphology are examined.

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Together with language, pictures have been connected to human culture from the very beginning (cf. [Schirra & Sachs-Hombach 2006a]). In the western societies they have gained a rather prominent place. However, steps toward a general science of images, which we may call ‘general visualistics’ in analogy to general linguistics, have only been taken recently (cf. [Sachs-Hombach & Schirra 2002], and [Schirra & Sachs-Hombach 2006b]). In computer science, too, considering pictures evolved originally along several more or less independent questions, which lead to proper sub-disciplines: *computer graphics* is certainly the most “visible” among them, but there are *image processing*, *information visualization*, and *computer vision*, as well. Only just recently, the effort has been increased to finally form a unique and partially autonomous branch of computer science specifically dedicated to images in general. In analogy to computational linguistics, the artificial expression ‘computational visualistics’ is used for addressing the whole range of investigating scientifically pictures “in” the computer (cf. [Schirra 2005]).

Pictures have to be formalized digitally in an adequate manner when computer scientists are to work with them. It is mainly the relevant physical properties of the corresponding picture vehicle that have to be considered in that formalization: that is, the picture syntax. The present special issue of IMAGE deals with exactly that theme taking the specific, formalizing perspective of computational visualistics. It is also intended as the attempt to offer a clear and easily understandable summary of the state of the art of research on picture morphology in computational visualistics for picture scientists of the other disciplines.

1 Computational Visualistics

Computational visualistics gains its name from its two parent disciplines: “computational” refers to the rather young discipline of computer science. “Visualistics” brings into mind the even younger unified science of pictures: general visualistics. Computer science, the endeavor of studying scientifically computers and information processing, has two different roots determining its methodology. In some aspects, computer science is a typical *structural science* like mathematics and logic: their subjects are purely abstract entities together with the relations in between. Such entities far off of our living practice are at best linked to everyday life by means of an interpretation relation arbitrary to the structures as such. With respect to some other aspects, computer scientists are like electrical engineers interested in *engineering* problems, an interest resulting in concrete artifacts that have already changed our lives dramatically during the past few decades and continue to do so with growing acceleration.

Correspondingly, the topics of computer science are, on the one hand, certain forms of purely abstract structures underlying data processing,¹ and on the other hand, certain kinds of purpose-bound artifacts we usually call “computers”. The concept “implementation” relates those two poles.²

¹ The processing of data is certainly a crucial theme for computer scientists, but it depends completely on the fact that data is always structured and grouped into types. Each such type implies a set of possibilities to “do something” with that kind of data: numbers can be added or multiplied (etc.); polygons in a geometric model can be moved or turned, mirrored or strained (etc.), but not *vice versa*. Usually, several data types and their interactions are relevant. As it is only important here that we can perform some operations with one sort of data so that certain relations hold between their results while ignoring the concrete manner of how those operations are actually realized, computer scientists consider *abstract data structures* – abstract entities that grasp exactly the essential properties. Algebraic formulae or logical expressions are often used to that purpose: the former for describing which operations transform the instances of which data type of the structure into what other type’s instances; and the latter determining which properties remain unchanged – invariant – after a certain sequence of operations; cf., e.g., [Ehrig & Mahr 1985].

² An implementation of an abstract data type – which is determined by a specification (a description) of its essential features – is a combination of more elementary data types, which are assumed unproblematic, so that the specification of the data type implemented is satisfied. If the data types employed for implementing are realized physically (e.g., in electro-technical devices), the implementation schema acts also as a plan for constructing a physical realization of the implemented data type. With such a physical realization, corresponding algorithms can be used to concretely manipulate instances of the data type. The implementation relation resembles the particular argumentation form expressed in synthetic judgments a priori by Kant (Critique of Pure Reason): In the mere specification, the essential features of an abstract data type remain contingent – like the axioms of a calculus. The implementation enables us to *found* those features: They are like

1.1 The Relation between Computer Science and General Visualistics

Quite obviously, *pictures* are not mentioned so far as a genuine topic of computer science. So, how are they linked with abstract data structures and their implementations on computers? That question is indeed a particular version of the more general problem of the relation between computer science and any domain of application; a relation that can be explained by means of the philosophical theory of rational argumentation (cf., e.g., [Ros 1999]) because the function of abstract data structures is equivalent to the function of the concepts structuring the rational argumentations in the domain of application. Data structures determine how formal expressions can correctly be constructed and transformed. The interrelated concepts that form a whole field of concepts³ – computer scientists sometimes use the expression ‘ontology’ in this context, as well – determine how we ought to speak in a rational manner about a certain thematic domain, for instance about pictures, and how we may draw correct conclusions from corresponding assertions (in general visualistics, in our example).

The relation between computer science and any domain of application employs that equivalence. Applications of computer science to a certain subject are mediated essentially by means of a *formal translation* of the field of concepts that structures the rational argumentations in the application domain under investigation into a corresponding abstract data structure. Computational visualistics can thus be characterized by means of its central topic: the data structure(s) »image« that can be conceived of as the formalized equivalent(s) of the field(s) of concepts that form(s) the subject of general visualistics; or in other words: the former ruling formal expressions that are correctly constructed and transformed if and only if they correspond to the latter, which determine how we ought to speak in a rational manner about pictures. Algorithms in those data structures exemplify potential argumentations in picture theory in a formalized manner. Therefore computational visualistics is indeed able to contribute, as well, to general visualistics in return: with its algorithms implemented, the results of applying a theoretically proposed argumentation in a formalized and automatized manner onto concrete examples can be demonstrated and examined in great number with dramatically reduced effort. This is particularly evident in a range of picture phenomena that would even not exist without the help of computers: the interactive images.

that, *because* they are implemented in a specific manner on those data types with their particular features; cf. [Schirra 2005, Sections 2.1 & 4.3.1.2].

³ If we refer by the expression ‘the concept »X«’ – e.g., by ‘the concept »image«’ – to everything that is structurally common to all explanations of ‘X’ (in the example: the expression ‘image’) and its synonyms [Wittgenstein 1953] – that is, everything that “remains the same independent of how or in what language I formulate or show it” – then naturally, we never examine one concept alone: it is always a system of concepts that are mutually related and cannot be defined independently from each other, like »king«, »queen«, »knight, and »medieval society« (or alternatively »chess«) or, of course, »image« and »perception«. They belong to the same *field of concepts*.

1.2 Components of Computational Visualistics

Most of the pre-existing picture-related subjects in computer science focus on only certain aspects of the data structure »image«. In the area called *image processing*, the focus of attention is formed by the operations that take (at least) one picture (and potentially several other parameters that are not images) and relate it to another picture. With these operations, we can define algorithms for improving the quality of images (e.g., contrast reinforcement), and procedures for extracting certain parts of an image (e.g., edge finding) or for stamping out pictorial patterns following a particular Gestalt criterion (e.g., blue screen technique). Compression algorithms for the efficient storing or transmitting of pictorial data also belong into this field.

Two disciplines share the operations transforming images into non-pictorial data types. The field of *pattern recognition* is actually not restricted to pictures, but it has performed important precursory work for computational visualistics since the early 1950's in those areas that essentially classify information in given images: the identification of simple geometric Gestalts (e.g., "circular region"), the classification of letters (recognition of handwriting), the "seeing" of spatial objects in the images or even the association of stylistic attributes of the representation. That is, the images are to be associated with a non-pictorial data type forming a kind of description. The neighboring subject of *computer vision* is the part of AI (Artificial Intelligence) in which computer scientists try to teach – loosely speaking – computers the ability of visual perception. Therefore, a problem rather belongs to computer vision to the degree to which its goal is "semantic", i.e., the result approximates the human seeing of objects and their behavior in a picture.

The investigation of possibilities gained by the operations that result in instances of the data type »image« but take as starting point instances of non-pictorial data types is performed in particular in *computer graphics* and *information visualization*. The former deals with images in the closer sense, i.e., those pictures showing spatial configurations of objects (in the colloquial meaning of 'object') in a more or less naturalistic representation like, e.g., in a computer game. The starting point of the picture-generating algorithms in computer graphics is usually a data type that allows us to describe the geometry in three dimensions and the lighting of the scene to be depicted together with the important optical properties of the surfaces considered. Information visualizers are interested in presenting pictorially any other data type, in particular those that consist of non-visual components in a "space" of states: in order to do so, a convention of visual presentation has firstly to be determined – e.g., a code of colors or certain icons.

1.3 The Concept »Image«

The central issue of computational visualistics depends, in conclusion, on the core topic of general visualistics, i.e., the concept »image«. Correspondingly, determinations of that concept in image science are highly relevant for structuring the investigation of the data structure »image«, its algorithms, and the implementations thereof. It may therefore be rather helpful to end this section about computational visualistics with a short note on the concept »picture« in general visualistics.

Unfortunately, picture science has not yet come to final conclusions concerning the complete “ontology”⁴ of pictures, which might be taken as the ultimate reference point for computational visualistics. Nevertheless, a sufficiently comprehensive determination to guide computer scientists dealing with pictures is available with Sachs-Hombach’s [2003] proposition of a general conceptual framework, namely to determine the concept »picture« as »perceptoid signs«.⁵ In the form of an Aristotelian definition with *genus proximum* (»sign«) and *differentia specifica* (»perceptoid«), this determination refers not only to two core aspects of pictures but opens originally, as we shall see below, the way to speak about pictorial syntax and picture morphology.

The superimposed concept »sign« implies that something – the *picture vehicle* – can be a picture if and only if it is in a certain way part of a special kind of situation that is characterized by a particular action: the sign act. That context also includes acting subjects called “sender” and “receiver”. The sign (e.g., a picture) is used by the sender as a means to direct the focus of attention of the receiver onto something that is usually not present in that situation.⁶

Furthermore, in order to function properly each picture has to apply our abilities of visual perception in a specific manner, which we call its »perceptoid« character. More precisely, in using – i.e., adequately using – pictures we do not only perceive visually the sign in its physical appearance, that is, the picture vehicle. We have also to invoke – at least to some degree – our abilities to visually perceive spatial objects and configurations that are closely related with what the picture is employed to symbolize (the picture content).

⁴ The term “ontology” is used here as in the context of computer science, i.e. equivalent to “field of concepts”.

⁵ The original German expression is “wahrnehmungsnahe Zeichen”, cf. [Sachs-Hombach 2003, Sec. I.3].

⁶ More precisely: sender and receiver are to be conceived of as roles that can also be simultaneously embodied by a single person. Correspondingly, we are able to bring something absent back into our own mind (and hold it in the focus of our attention) by means of “presenting a picture of it to ourselves” – only we say then plainly that we “look at the picture”.

2 Pictures and Syntactic Investigations

Taking pictures to be a kind of sign allows the visualists – and that is, the computational visualists, too – to apply semiotic distinctions in order to guide their investigations. Since a picture like any sign depends on being part of a sign act, the broadest range of investigations (enclosing and determining all other questions) is the one that examines any relations between the other acts of sender and receiver with the signing activity – i.e., the presentation of a picture by a sender to a receiver in a certain context. That is the field of *pragmatics*. Examinations considering only the relations holding between the picture vehicles and what they are used to symbolize for sender and receiver determine the field of *semantics*.

Syntax is the third semiotic range of questions; and it is also the most restricted one since it deals with the sign vehicles (or in our case: the picture vehicles) alone. More precisely, the classifications of and relations between sign vehicles with respect to their physical properties are examined. This also includes the question of the range of variability of sign vehicles that may be used as the *same* sign, but also potential compositions of sign vehicles to more complex sign vehicles.

2.1 Syntactical Density

Syntactical considerations belong to the repertoire of picture theories since Nelson Goodman's publications at the latest (cf. [Goodman 1976], and also [Sachs-Hombach & Rehkämper 1999]). Although Goodman does indeed consider more than syntax, it is an important syntactic characterization of pictures that has had the most influence in general visualistics, so far. Syntactically, pictures are, he proposes, *dense* – in contrast to verbal signs, which are *syntactically distinct*. A sign system is called syntactically dense if the dimension of values for at least one of the syntactically relevant properties of the sign vehicles corresponds to the rational numbers: between any two values there are always more values. Sign vehicles with different values in that property are taken as different signs in that sign system. So, two of the infinitely many signs of such a system can be "infinitely similar" to each other, as there are always more sign vehicles "in between".

Syntactic characteristics of pictures are obviously defined by the visual properties of a marked surface of the picture vehicle. There are at least two different relevant dimensions that are apparently dense: (i) the positions of a point of color or a border between colors, and (ii) the perceived color (in a broad meaning). Between two different positions of a point of color, there is always – at least in theory – a (multitude of) position(s) in between. And similarly, in the theories of color two different color values are always connected by means

of a sequence of intermediate color values, even if the human eye may not be able to distinguish those without the help of an artificial instrument.

The syntactically characteristic property of density is of high significance for the possibility of encoding, presenting, storing, and transferring pictures by computer. Is it decidable whether two pictures are syntactically equal? Can we, with other words, determine by means of effective, finite algorithms whether the transmission of a picture vehicle through the Internet, for example, has been correct, or whether a stored image still corresponds exactly to the original? Goodman has denied that possibility, which means that computational visualistics has a problem if he is right. Any computer system would only be able to differentiate picture vehicles up to a certain degree of resolution (in location or color).

2.2 *Resolution in Computational Visualistics: Pixels*

Indeed, the combination of images and computers did originally cost the former a property conceived of as characteristic for pictures by the scientists of many disciplines involved: pictures had to become digital in order to join that liaison. Essentially, 'being digital' means that the resolution of pictures has a definite (and often quite small) value. In contrast, the common view holds that picture vehicles have to be (at least in principle) analogous, i.e., without any limitation of resolution.

The most simple and well-known type for making picture vehicles available for a digital computer are bitmaps – matrices of *pixels* as they are called ('picture elements'). This data type allows us to define a pixel-value for any pair of coordinates taken from two finite sets of successive indices (i.e., natural numbers). The pixel values encode a visual property, like color or intensity. Bitmaps have therefore a finite and fixed locale resolution that depends on the size a pixel is given: bitmap pictures are ratcheted. The number of different bitmaps of a given matrix size is finite, while the number of different matrix sizes is infinite but enumerable.⁷

The presentation of pictures on a computer screen typically employs this data type in just one matrix size. Although only a finite number of different picture vehicles is discriminated in that manner, an underlying data structure »image« still can be designed in order to fit the criterion of syntactic density imposed by general visualistics: the dense structure of a picture has to be projected (potentially only in parts) onto the syntactically distinct pixel matrix with the option of zooming in and out. In contrast to the visual approximations shown on the screen, a picture encoded by an instance of a data structure incorporating such a zoomable projection function needs not having a finite level of resolution (at least in

⁷ Thus, although bitmaps are a rather limited candidate for the data type »image«, they have at least the advantage that there is no problem to decide identity or difference between two instances effectively.

theory: recall for example the small program systems fashionable few years ago that were used to visually inspect certain fractal functions, e.g., the Mandelbrot set).

Resolution is only one aspect of computational pictorial syntax: It corresponds roughly to the level of linguistics dealing merely with the range of letters; the notorious pixel usually comes into the beholder's (or creator's) focus of attention only when the presentation quality of a picture is low. There are other parts of which a picture vehicle is viewed as composed of and which could be rearranged to form another picture vehicle: When discussing syntactic design elements M. Scholz (1999), for example, refers to Paul Klee's pedagogic sketch book (1925, republished 1997) as an overview. Klee proposes several kinds of points, spots, lines, and areas (including typical geometric Gestalts like circle or square).⁸ We shall later come back to such entities from geometry. Sometimes, candidates for syntactic elements can also be defined based on the production process: each stroke of a pen, a brush or a graving tool may lead to an individually visible mark usable as a syntactic element.

Of course, confronted with the questions of pictorial syntax and its combination rules, the first impulse of computer scientists is usually: to think of formal grammars.

2.3 *Picture Grammars*

Every computer scientist knows by heart the abstract structures called formal grammars – also called Chomsky grammars or compositional grammars or transformation grammars – since those are the major instrument for defining and classifying linear structures like programming languages. They are actually a tool from linguistics and have been applied to verbal syntax with great success.

A compositional grammar provides (i) a finite set of grammatical categories like 'article', 'prepositional phrase' or 'sentence', (ii) a lexicon (i.e., a collection of basic signs (words) each associated to a grammatical category), and (iii) a finite set of composition (transformation) rules. Essentially, each rule associates a grammatical category with a sequence of such categories, like in the following examples:⁹

PP → Prep + NP

NP → Art + Noun

NP → Art + Adj + Noun

⁸ The major distinction in each of those element groups is that of an active, passive or medial element, depending on the role the design element plays in composition and production.

⁹ In the examples, the usual labels 'PP' for prepositional phrase, 'NP' for noun phrase, 'Art' for article, 'Prep' for preposition, and 'Adj' for adjective are used. The rules can be employed mechanically in two ways: first, a given combined sign – a sentence – can be analyzed: it belongs syntactically to the language determined by the grammar if the sequence of syntactic categories that are associated to the words forming the sentence can be projected backward by means of several transformation rules to a special syntactic category (usually called 'sentence' or simply 'S'); second, starting from 'S', a number of applicable combination rules is used in forward direction in order to synthesize a list of syntactic categories that can be associated to words in the lexicon generating a well-formed sentence.

Those three sets determine all sentences, i.e., sequences of words, belonging to the language considered. Note that each word listed in the lexicon always has clear semantic and grammatical functions of its own.

Assuming that all pictures form just one “language”,¹⁰ a formal grammar for picture syntax thus would also have to provide corresponding sets of syntactic categories, elementary pictures with associated syntactic categories, and composition rules. Those set should be accordingly applicable for analyzing in a mechanical manner given objects in order to decide whether they are pictures,¹¹ or to generate from the starting category any picture vehicle. Such a formal grammar for pictures would indeed enable us to distinguish between well-formed and ill-formed picture vehicles.

Unfortunately, all proposals so far to provide such a combinational grammar system for all pictorial signs (or even large subsets) have failed: only very special pictorial media – that apparently are also used in a way similar to language anyway, like pictograms – could be formalized in that manner.¹² In general, there does not even seem to be anything like an ill-formed picture vehicle at all (cf. [Plümacher 1999]). Any more or less flat surface that can be visually perceived can apparently serve as a picture vehicle.

Already the question “what are the syntactical elements in the ‘lexicon’ – as we do not have a better expression, so far – of copper engravings (for example)” is not easily answered. Can the engraving lines carry that function? Are pixels – as used in computer visualistics – better candidates? However, neither engraving lines nor digital pixels bear a proper pictorial meaning by their own – one of the characteristics in the linguistic case, i.e., for the words in the lexicon.

Furthermore: What corresponds to the grammatical categories? Are perhaps “Circle” or “Spot” pictorial analogies of “Noun” and “Art”? And if so, what would actually be the difference between the ‘lexical’ basic elements and the grammatical categories in that system?

In conclusion: Being rather fertile in linguistic syntax studies, the idea of generative syntax has often been proposed for pictorial syntax, as well – though, with little success: compositional syntax is mainly interested in the syntactically correct composition of words (as elementary verbal signs) into sentences (i.e., compound verbal signs). A pictorial analogy of words so that pictures could be conceived of as corresponding sentences has not been suggested in a convincing manner. However, another important building block of syntax studies – at least in linguistics – is given by morphology.

¹⁰ Alternatively, several pictorial subsystems may be syntactically distinguishable.

¹¹ – or belong to the particular pictorial subsystem in question.

¹² Similarly, arrangements of pictures as in journal layout or comics, and even the sequences of scenes in film can partially be analyzed by means of formal grammars.

3 Morphology

3.1 Morphology in Linguistics

In linguistic morphology, the rules of building words, and hence the inner structure of words is examined instead of sentences.¹³ Words are partitioned in segments called ‘morphemes’¹⁴ that contribute to the word’s meaning or grammatical function. The postfix ‘-ed’ in English, the prefix ‘pré-’ in French, or the root ‘-wend-’ in German are typical examples for morphemes. Mostly, morphological elements are identified and arranged into classes by means of a rule of mutual exchange: some words beginning with ‘pré-’ can be transformed into other words of French by just changing the prefix to ‘re-’, ‘con-’, ‘de-’ etc.

More generally, morphological modifications can be differentiated into internal modifications mainly by means of vowel permutations (e.g., ‘come’ to ‘came’), and external modifications by means of affixes – beside prefix and postfix, some languages also use infix and circumfix modifications. While inner modification alters the “color” of a word, so to speak, external modification changes its shape and size. Thus, the combination of morphological elements also plays a major role in the invention of completely new words.

Morphemes do not have to be – and are usually not – words by themselves.¹⁵ Even the semantic or grammatical function of one morpheme can be ambiguous and may change in different compositions (e.g., “s” as flexion postfix and plural postfix in English). Morphemes may best be viewed as the vehicles of unsaturated partial signs acts without an independent pragmatic function¹⁶ that modify in a more or less specific way the meaning of the whole.

There are arguments that syntax in the form of a formal grammar, and syntax as morphology are not categorically opposed but form the two ends of a more or less continuous scale of various language structures: from the *analytic* language structure (also: isolating languages) to the various types of *synthetic* language structures (with the subsets of agglutinating, flexing, and fusing languages), and finally the *polysynthetic* language structure (also: incorporating languages).¹⁷ In an extremely isolating language (like Chinese), words are never ever modified. All grammatical relations are expressed by special words. Sen-

¹³ Although the expression “morphology” was already introduced to linguistics by August Schleicher in 1859 (under the influence of Goethe’s morphological theory of plant growth), a specific morphological investigation of words – in contrast to syntacto-grammatical studies of sentences and apart from phonology – did not become prominent before the 1970s.

¹⁴ The term “morpheme” was proposed around 1881 by B. de Courtenay and elaborated by L. Bloomfield.

¹⁵ Morphemes that are also words are called *free*; the other morphemes are *bound*. As free morphemes are listed in the lexicon, they are also called *lexemes*.

¹⁶ This is in contrast to predication or nomination, which are also unsaturated partial sign acts but each carrying a quite specific pragmatic function (of introducing a distinction to the discourse universe, or naming a discourse object respectively).

¹⁷ The distinction was already introduced by W. v. Humboldt and A. W. v. Schlegel.

m a t g ī b u l h a h u m š

		(<i>gāb</i>) <i>gīb</i>		
	<i>t(i)</i>	...	<i>u</i>	
			<i>l(ī)ha</i>	
				<i>hum</i>
<i>ma</i>	...			<i>š</i>

Schema 1

tences are straightforward groupings of words usually in a relatively strict word order; correspondingly, a separate study of morphology does not make sense. An extremely polysynthetic language would in contrast consist of one-word sentences only, a single word that may consist of many morphemes all melted together in order to modify the complete meaning accordingly. Thus, syntactical investigations here are purely morphologic.¹⁸

The morphological structure of the word *matgībulhahumš* in Egyptian Arabic,¹⁹ for example, could be literally translated to approximately “not-you-all-ought-bring-her-them-thing” (i.e., “do not bring them to her, all of you”). It consist of the two circumfixes *ma...š* (“not ... thing”) and *t(i)...u* (marker for 2nd person plural imperfect in jussive mode: approx.: “you ought to”), the two morphemes *l(ī)ha* (3rd person singular feminine dative), *hum* (3rd person plural accusative), and, as the root, an internally modified lexeme *gīb* (the imperfect form of *gāb*: “to bring”), as is indicated in schema 1.

All the morphological elements are fused to a single word that is used as a sentence. The schema of such complicated combinations by means of the fusion of morphemes with partial phoneme elisions – together with the used of enclosing or inserting affixes – can indeed much stronger evoke the idea of a syntactic structure of pictures than the schema of formal grammars.

3.2 *Transfer to Visualistics*

Intuitively, the system of pictures and most of its subsystems are similar to extremely polysynthetic languages. Of course, picture vehicles do have parts that modify the pictorial meaning and use of that vehicle. But for each picture, those parts are closely fused together – comparable to an enormously complex one-word sentence. They form a single

¹⁸ Many Native American languages like Náhuatl are more or less strongly incorporating. Flexing and agglutinating languages are somewhere between the two extremes. The word order is usually not as restricted as in isolating languages, and a mixture of grammatical and morphological rules determines the syntactic structure.

¹⁹ Linguists report that the Egyptian version of Arabic has a strong tendency to polysynthetic structures in contrast to high Arabic.

entity that does not allow us usually to isolate in a clear manner the semantic contribution of any part, as it had to be expected in the case of a formal grammar. Nevertheless it is clear that any morphological element of the picture vehicle – or *pixeme* for short – does contribute in some way to the meaning, and hence modifies the use of the picture. Therefore, any tiny change in the spatial distribution of pigments may very well be seen primarily as a modification in the sense of morphology.

There are several characteristic differences to verbal morphology: In contrast to the essentially temporal and hence linear composition of verbal morphology, pictorial morphology extends in (flat) space and thus in (usually) two coordinated dimensions, which increases the complexity quite heavily. Instead of the pair of possible directions for morphological extensions – “before” (as prefix) and after (as postfix)²⁰ – an infinite and actually dense multitude of directions can be used to position pixemes.

Of course, the specific difference of resolution already mentioned above has to be taken into account, as well: there is a distinct lower limit to resolution in linguistics since morphemes cannot be smaller than the difference between two letters or phonemes. For picture vehicles, no such quantization is evident. The criterion of density also implies that any pixeme can – at least in principle – be considered as composed of even smaller pixemes.

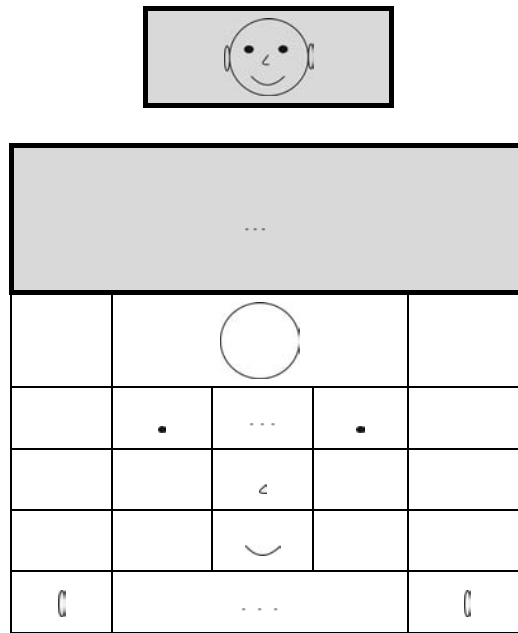
Brush strokes, pencil lines, etc. are rather good candidates for simple pixemes, as was already mentioned above.²¹ They are composed into more complex configurations that nevertheless still are pixemes. In general, we may view any geometrical entity of two-dimensional geometry of the picture vehicle as a pixeme. Then, even a picture is a pixeme, as well – which makes sense as its surface can be seamlessly incorporated in another picture vehicle. Still, pictures may very well have a morphological structure without a list of given elements that are pictures themselves. Although there appears to be no (natural) verbal language that employs bound morphemes only, morphology does not necessarily depend on the existence of free morphemes (lexemes). On the other hand: there always exists a maximal pixeme to which all the other pixemes are infixes. It is the frame that externally binds and thus determines the maximal pixeme. Indeed, maximal pixemes might act as free morphemes for picture vehicles.²² While verbal structures grow morphologically outward by adding elements mostly externally, pictorial structures grow morphologically inward by adding details internally.

Since morphemes essentially change the color of vowels in the course of an internal modification, a literal change of color of a pixeme is a very plausible candidate for the corre-

²⁰ Circumfixes employ accordingly both directions, and infixes can be seen as inverse circumfixes.

²¹ See also the contributions of Engelhardt and of Isenberg in this volume.

²² As another hint for a kind of free pixemes the following psychological evidence may be counted: a schema corresponding to an elementary face pixeme (or rather a set of affect-expressing face pixemes) is inborn to all human beings and already effective for very young children.



Schema 2

sponding derivation. Again, the bandwidth of alternatives is characteristically different: a finite set of phonemes vs. the colors from a dense range of options.

Evidently, the rules of visual perception are constitutive for the “segmentation” of pictures in pixemes.²³ The empirical findings from psychophysics and the concepts of Gestalt theory in particular help to determine the laws of pixeme formation. The former indicate general principles of indiscernibility of optical properties while the latter formulates grouping principles that bind compound pixemes to the constituting simpler pixemes. That decomposition runs down to optically uniform regions, which we find on any level of resolution since we deal with dense fields both in color and in location. An optically uniform region is not only given by a single color, but also by a color gradient (in particular a saturation or intensity gradient), and even by homogenous textures.

As an extremely simplified example in analogy to the verbal example above, schema 2 exemplifies a morphological (de)composition for a picture. In accord with the assumption mentioned above that pictorial morphology grows inward, the frame defines the root in the decomposition, or more precisely: bound by the frame, the empty “canvas” acts as the maximal pixeme. As an infix, the face marks modify the maximal pixeme. The face mark itself consists of a simple circular pixeme with several infixes and one circumfix (the ear marks).

Of course, the specific difference of resolution already mentioned above has to be taken into account, as well: there is a distinct lower limit to resolution in linguistics since morphemes cannot be smaller than the difference between two letters or phonemes. For pic-

²³ See also the contributions of du Buf and Rodrigues, and of Hermes and the SVP Group in this volume.

ture vehicles, no such quantization is evident. The criterion of density also implies that any pixeme can – at least in principle – be considered as composed of even smaller pixemes.

Although there is still much more to be said about pictorial morphology in general, it is now the time to come back to the particular perspective of computational visualistics.

4 Aspects of Picture Morphology in Computational Visualistics

Morphological considerations in the particular context of computational visualistics are at the focus of this thematic part of IMAGE V. We are interested in questions like the following: What alternative formalizations for pixemes apart from pixels can be offered by computational visualistics? Where and in which form do such formal pixeme systems play an important role? And what is the influence these formalizations in computational visualistics have on picture morphology in general?

4.1 Some Specific Approaches

Let us concentrate for the moment on lines or strokes. A stroke may be defined pragmatically by the painter's movement or semantically as the contour line of an object. Beside the potential graphical meaning of a line or the stylistic indications associated with its particular make (not to mention any other expressive or appellative function of dynamism associated to it on the level of pragmatics), there are several dimensions in which a line – just being taken as a line – can vary: most prominently in the course or path it takes. But there are other ranges: is it a continuous line, or dashed, or dotted? Does it consist of strokes of one kind or another? How thick is it? Does its thickness change over its course or not? Is there an internal fine structure to the strokes?

An extensive treatment of data types for strokes and lines and their possible implementations has been performed in the context of non-photorealistic rendering (NPR), a sub field of computer graphics.²⁴ While Figure 1 exemplifies several types of digital “hairy brush strokes” that have been generated – quite expensively in computational resources – by simulating a brush with several individual bristles applied with changing pressure to a certain kind of surface, Figure 2 shows examples of lines resulting the application of a “style function” to the “skeletal path” of the stroke.²⁵ Both constituents of the latter case are defined by means of parametric curves: the style describes how a given path (as the core of the line) is to be perturbed in order to result in a corresponding pixeme. Style and path can be viewed as independent ranges determined in each particular picture by semantic and / or pragmatic aspects.

²⁴ See also the contributions of Isenberg, and of du Buf and Rodrigues in this volume.

²⁵ Figure 1 was quoted from [Strassmann 1986], Figure 2 from [Schlechtweg & Raab 1997].



Figure 1: Enlarged Fine Structure of Computer-Generated Stroke Types

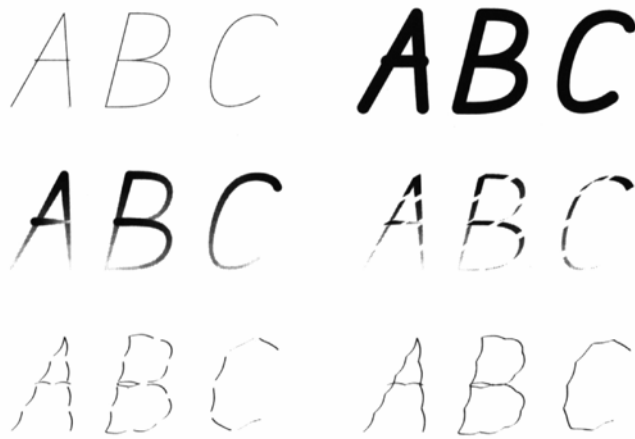


Figure 2: Examples with Style-Parameterized Stroke Functions

To some degree, the rules of composition of strokes or other pixemes into a picture can be investigated by means of the tools of formal languages. Formal grammars based on replacement rules that lead to two-dimensional “pictorial” structures have been investigated essentially under the name of L-systems. The expressions generated by an L-system can be interpreted as orders to place substructures, and to move or turn in-between. A fairly simple example is defined by the following replacement rule:

$$P \rightarrow P[-P]P[+P]P$$

Interpret “P” as “place a pixeme and move a bit forward”, “+” by “turn right”, “–” by “turn left”, and the square brackets as stack operations that allow us to return to that point after the bracketed sub expression has been dealt with. The plant-like structures in Figure 3 have been generated by this rule. Obviously the pixemes themselves are not really relevant for L-systems and their relatives, since these grammars basically deal with arrangements and groupings of abstract entities that may or may not be interpreted in a pictorial sense.²⁶

For a more extensive approach to pictorial morphology, a data type for pixemes can best be derived from a calculus for geometry. That any pixeme must be a geometric entity seems almost too trivial to be mentioned. That inversely any entity in flat geometry – apart from non-extended points – may also be a candidate for a pixeme is at least a good guess. Taking the common Euclidean formalization of geometry leads however to the “unpleasant” consequence that the most basic pixemes must be non-extended points – a concept highly abstracted from experience, that is.²⁷

²⁶ See also the contribution of Kurth in this volume.

²⁷ See also the contribution of Engelhardt in this volume.

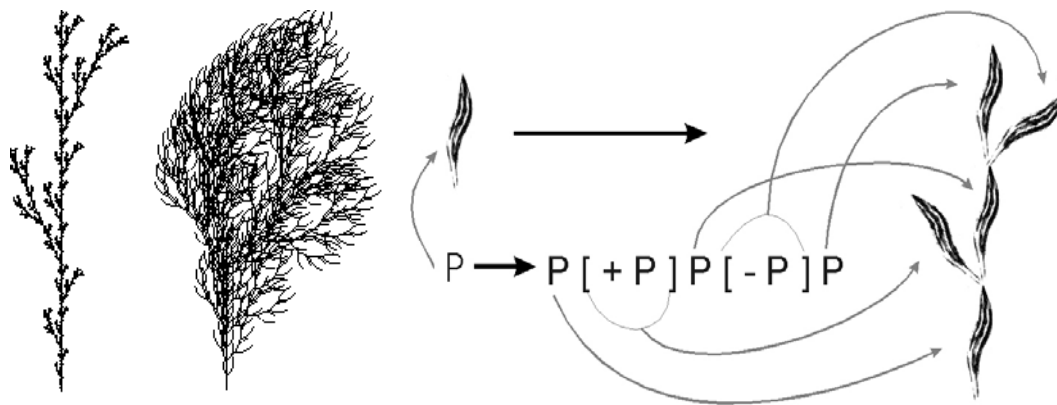


Figure 3: Two Complex Example Morphemes Generated by (Bracketed) L-Systems, and the Graphical Interpretation for the Rule for the Left Example

Fortunately, some non-standard approaches to geometry offer an interesting way out. The traditional calculus of geometry develops around the fundamental concept of a zero-dimensional point. In contrast, the family of *mereogeometries*²⁸ is based on extended regions as the most elementary entities, which may or may not have (distinguishable) proper parts. The regions are often called “individuals”. Individuals do not have immediate attributes of form or position: only the relations to other individuals, in particular parts, determine form and (relative) location.

An individual may quite well be thought of as a visual Gestalt – thus following the principle of perception psychology of the Gestalt school: one has to consider the perceived whole first and introduce the concepts for perceptual atoms as instruments of the explanations of the former, not the other way round. We do not see sets of zero-dimensional points but regional Gestalts. The abstract notion of a spatial entity without extension is secondarily constructed in order to explain some aspects of experienced space, but leads on the other side to severe difficulties as the discussion on infinite resolution has shown. Therefore, the constructs of an individual calculus for the two-dimensional mereogeometry are excellent candidates for a general and exhaustive discussion of pixemes.

In fact, the concept of a minimal region can be introduced in mereogeometry: They are usually called a “point”, but we may well use “pixel” instead. A point in this sense is a region that has no proper parts (or rather, a region where no proper parts are considered). When the concept »point« is introduced in the data structure in that manner, there is no need in any concrete instance for using infinitely many point instances: only the “relevant” points must be instantiated. This also means that there is always a finite resolution.

²⁸ See also the contribution of Borgo *et al.* in this volume.

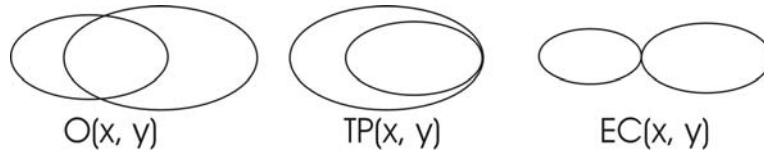
Mereogeometrical Calculi of Space

Mereogeometries are a group of particular logical formalisms for describing n-dimensional space that are based on mereological calculi. Mereology's focus of interest is part-whole relations. For instance, Clarke's mereological calculus (1981; here quoted from Vieu 1991, 120ff) is based on the primitive relation $C(x, y)$ with the intended meaning "individual x is connected with individual y " and defined by the axioms:

A0.1	$\forall x (C(x, x) \wedge \forall x \forall y (C(x, y) \Rightarrow C(y, x))$	Reflexivity and symmetry
A0.2	$\forall x \forall y (\forall z (C(z, x) \Leftrightarrow C(z, y)) \Rightarrow x=y)$	Axiom of extension

Some definitions possible in Clarke's calculus are ...

D0.1	$DC(x, y) \equiv_{\text{def}} \neg C(x, y)$	" x is disconnected with y "
D0.2	$P(x, y) \equiv_{\text{def}} \forall z (C(z, x) \Rightarrow C(z, y))$	" x is part of y "
D0.3	$PP(x, y) \equiv_{\text{def}} P(x, y) \wedge \neg P(y, x)$	" x is a proper part of y "
D0.4	$O(x, y) \equiv_{\text{def}} \exists z (P(z, x) \wedge P(z, y))$	" x overlaps y "
D0.6	$EC(x, y) \equiv_{\text{def}} C(x, y) \wedge \neg O(x, y)$	" x is externally connected with y "
D0.7	$TP(x, y) \equiv_{\text{def}} P(x, y) \wedge \exists z (EC(z, x) \wedge EC(z, y))$	" x is a tangential part of y "
D1.1	$x=F(\alpha) \equiv_{\text{def}} \forall y (C(y, x) \Leftrightarrow \exists z (z \in \alpha \wedge C(y, z)))$	" x is identical to the fusion of the set of individuals α "
D1.2	$x+y \equiv_{\text{def}} F(\{z : P(z, x) \vee P(z, y)\})$	" $x+y$ is the sum of x and y "
D1.5	$x \wedge y \equiv_{\text{def}} F(\{z : P(z, x) \wedge P(z, y)\})$	" $x \wedge y$ is the intersection of x and y "



... so that, for instance, the following theorem can be proven:

$$T0.34 \quad \forall x \forall y \forall z ((TP(z, x) \wedge P(z, y) \wedge P(y, x)) \Rightarrow TP(z, y))$$

A definition of "point" out of a set α of individuals (with Λ being the empty set):

$$PT(\alpha) \equiv_{\text{def}} \neg \alpha = \Lambda \wedge \forall x \forall y ((x \in \alpha \wedge y \in \alpha) \Rightarrow (EC(x, y) \vee (O(x, y) \wedge x \wedge y \in \alpha))) \wedge \forall x \forall y ((x \in \alpha \wedge P(x, y)) \Rightarrow y \in \alpha) \wedge \forall x \forall y (x+y \in \alpha \Rightarrow (x \in \alpha \vee y \in \alpha))$$

(i.e., all individuals partaking in a point are connected with each other; if two of them overlap, their intersection is also part of the point; each individual containing an element of the point is also element of that point; and finally, if an element of the point is the sum of two individuals)

This calculus already allows dealing with topological relations and can be extended easily to a full geometry (i.e., including directions and metric distance). That is, any geometrical configuration can be described by a set of propositions of that calculus. Any analysis or transformation of the geometrical configuration can correspondingly be performed in analogy with the set of propositions by means of logical analyses or transformations (cf., e.g., Pratt-Hartmann 2000).

While Euclidean geometry first introduces the continuous range of infinitely many coordinates determining potential points some of which are then chosen to be relevant (still an infinite number in any practical relevant instance), mereogeometry starts with a (usually finite) number of relevant individuals (regions) we can think of being given in perception. That is, we may indeed assume that the principles governing visual perception determine the regions that are syntactically relevant, hence leading only to the essential “points” determined by the given individuals.

The empty picture plain – as the simplest maximal pixeme – is particularly characterized in its most usual rectangular form by the four corner points. The “energetic field” often associated to such a maximal pixeme (cf. Fig. 4) cannot easily be derived as it depends essentially on features of the perceptual mechanism not covered by the Euclidean calculus as such.²⁹ Additional explanations have to be added that often employ rather mystical metaphors to physics.³⁰ The mereogeometrical conception of points and limits may offer a better access to the problem of the “energetic aspects” of pixemes, and especially of the empty picture plane: As those points are only conceivable as the result of operations on extended regions, the four corner points implicitly refer to defining individuals (virtual pixemes). It is a promising hypothesis for future research to derive within the calculus of mereogeometry any “energetic effects” from those implicit pixemes.

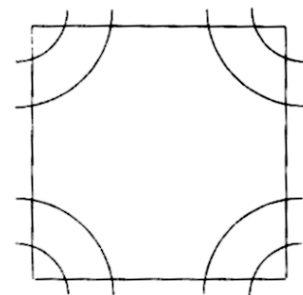


Figure 4: Rectangular maximal pixeme with “energetic phenomena” as sketched by Saint-Martin [1990, 97]

Mereogeometries are a formal way to deal with geometry in a manner more closely related to visual perception than traditional point geometry. If we accept the view that the central data type of a two-dimensional mereogeometry determines what is a pixeme – namely any connected sub system of individuals, then there is indeed no finite number of possible pixemes – a clear difference to verbal sign systems with their strictly limited number of morphemes. However, any pixeme can be described and dealt with in a unique and generatable manner in the calculus in a finite number of steps: pixemes can be combined to form pixemes of a higher order – until every visually separable Gestalt of a picture is covered.

²⁹ cf. Saint Martin 1990, 96: “By reason of its dynamic origin, this Basic Plane must be defined as an energy-charged portion of space, generated by the radiating energies produced by the angular intersections of the four straight lines. It is through this maximal energizing of right angles that a dynamic structure emerges and is propagated to form a Basic Plane. Irrespective of the physical characteristics of the material support which facilitate its deployment, the Basic Plane is defined as an ensemble of energetic phenomena, taking its point of origin in the peripheral lines and corners that envelop and contain it. Its energetic and topological characteristic will remain the essential element which determines the spatial structure of the Basic Plane”.

³⁰ *ditto*, p. 97: “While essentially describable as the interplays of various levels of intensity of energy, perceptual systems are animated by the different categories of actual, potential, and virtual energies offering a decreasing order of forces. The actual and potential levels are established by the contribution of both the visual elements and perceptual processes, the virtual being the unique product of perceptual activity”.

4.2 *The contributions of the volume*

Since the fluctuation of the focus of attention between structural science and engineering is characteristic for all investigations in computer science, it is also valid for the dealing with pictorial data. On the one hand, particular abstract data types for pictorial representations are investigated and designed from a purely structural point of view. For example, efficiency properties are examined, or minimal sub-structures for particular tasks determined. On the other hand, concrete algorithms for, e.g., picture processing are “software-engineered” and used in diagnosis. Correspondingly, the papers collected in this issue exhibit a wide range between analytic investigations and constructive engineering.

The call for paper for this thematic issue of IMAGE did in particular list the following five ‘crystallization cores’ for a discussion of picture morphology from the perspective of computational visualistics:

- Picture morphology as Grammar: L-Systems and Similar Formalizations
- Mereo-Geometrical Approaches to Picture Morphology
- Pixemes in Non-Photorealistic Computer Graphics
- Image Processing: Pixeme-based Approaches of Picture Manipulation and Computer Vision
- Glyphs and Icons: Pixemes in Information Visualization

With the exception of the fifth theme – each item has been covered by at least one contribution.

The first two texts deal with the general question of the systems of pictorial syntax or morphology and its constituents. A set of building blocks for formally describing graphics is presented in the contribution of **Engelhardt** (Netherlands). He takes a perspective rather related to design and design theory, and proposes a set of building blocks for all graphics derived from the relevant literature. Three types of building blocks are distinguished: graphic objects, meaningful graphic spaces, and graphic properties. Although this system does not yet reach the formal stringency of the logical calculi employed, for instance, in the formal ontology of space, it provides a good entry point for the discussion of computational picture morphology.

An overview on the formal representations of space in the field of formal ontology, a sub-domain of AI and cognitive science, is given by the contribution of **Borgo** and colleagues (Italy). Without putting too much stress on the (actually rather demanding) underlying logical and mathematical formalizations, these authors explain the advantages of mereo-geometrical approaches in the cognitive dimension fitting the qualitative categorizations of the human access to space. From that perspective, they discuss the application of mereo-geometrical calculi to the description of pictorial morphology. While Engelhardt starts from more or less informal notions as used in design theories and proposes a systematic cate-

gorization of graphic objects, rules for their combination, and a typology of meaningful graphic space, the Italian group moves from highly formalized concept (which are elaborated in formal calculi) toward the more informal notions employed in pictorial syntax.

With his survey on morphological models with L-Systems and relational growth grammars, **Kurth** (Germany) brings into the debate another meaning of the expression ‘morphological’ – a meaning more closely related to the word’s original, i.e., biological context: the knowledge about the bodily forms of living beings, and the rules of the arrangements of body parts and organs (especially in the temporal development). The special grammatical formalisms described by Kurth do not originally refer to pictures but to objects that are conceived of as being constructed by formally arranging parts in space by means of a quasi-biological manner of “growing”, and that are often depicted in order to be further studied or used. Therefore, this meaning of ‘morphological’ actually exceeds the borders of strict syntax – after all, the structures of the things depicted are actually in the range of semantics. Nevertheless, the formal options given by means of quasi-grammatical mechanisms for “growing” spatial arrangements of “body parts”, and the geometrical arrangement of pixemes are close enough to the discussion on pictorial morphology for further enlightening the latter.

While Kurth is more interested in the arrangement, i.e., the spatial configuration of any kind of parts, the contribution of **Isenberg** (Canada), turns our focus of interest to the potential parts to be arranged by giving an overview on the techniques used to generate computer graphics apart from naturalistic – say: “photo-realistic” – representation styles. Contrasting the resulting images with the photorealistic case, Isenberg describes a wide range of morphological modifications possible with those techniques. Different rules for calculating shading, for example, lead to a picture that is internally modified; applying strokes or graftals corresponds to external modifications. Unlike the pixel, the morphological primitives used in NPR often carry a “meaning” beyond the syntactical level; saving the morphological structure with the picture is therefore, so Isenberg, often quite helpful for subsequent processing.

The paper of **du Buf** and **Rodrigues** (Portugal) also aims for non-photorealistic rendering, as the authors explain how computational models of neuro-physiological explanations of visual perception can be employed in order to generate painterly pictures. After giving us an overview about the relevant state of the art of neuro-physiological analyses, they consider the relation between bottom-up processing (pixels to higher pixemes) and top-down projections (from semantic entities to pixemes), and sketch a computational model of the visual system that can systematically re-create a visual input in the form of a painting.

A strictly engineering perspective is finally taken in the text of **Hermes** and **the SVP Group** (Germany), which also broadens the view to moving pictures: how can an accept-

able movie trailer – as a kind of cinematic summary – be more or less automatically generated on basically syntactic principles from the movie. In the practical point of view, the theoretical discussion about the elements of picture morphology retreats behind the complicated concrete problems at hand. Due to that complexity, the task has even to be restricted to a certain genre (and certainly bound by the current “taste” for trailer esthetics). The focus is mainly on “shots” and the transitions between them. The group presents a program system, the outcome of which has been empirically compared with satisfying results to commercial trailers produced in the ordinary way. In contrast to the neuro-physiologically inspired analysis of an input picture in the contribution of du Buf and Rodrigues, the input movie is analyzed with several standard techniques from computer vision like motion-based segmentation, and face detection and recognition (supplemented by a range of classification/recognition methods for acoustic input or even text) – techniques that are not necessarily cognitively adequate but basically optimized for the tasks they have to solve. Unlike the system of du Buf and Rodrigues, the final (re)creation of a (moving) picture depends on a separate set of templates following semantic and pragmatic aspects.

As the thematic issue of IMAGE on computational image morphology attempts in particular to mediate between computational visualistics and other disciplines investigating pictures and their uses, a final chapter broadens the perspective again and relates the computational argumentations of the preceding papers to the more general discussion of image science. I, then, also extend the discussion to the question of syntactically ill-formed pictures and the limits of pictorial syntax or morphology.

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